

Modeling the Clustering of Objects

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Abstract.

I review the main steps made so far towards a detailed (semi) analytical model for the hierarchical clustering of bound virialized objects (i.e., haloes) in the gravitational instability scenario. I focus on those models relying on the spherical collapse approximation which have led to the most complete description. The work is divided in two parts: a first one dealing with the mass function of objects and a second one dealing with the growth times and rates.

1. Introduction

Observational data of cosmological relevance refer to the characteristics of the CMB radiation and the clustering and structural properties of bound virialized (more exactly, relaxed and steady) objects such as Lyman- α clouds, galaxies, and galaxy clusters. To take full benefit of the information contained in the latter kind of data a good knowledge of how, when, and where these objects formed and evolved is required. Indeed, this would allow us not only to correctly interpret the observed properties of those cosmological objects but also to properly use them to constrain the correct cosmogony (i.e., the possible Gaussianity of the initial density field, its power spectrum, and the values of Ω , Λ , and H_0). Unfortunately, the modeling of the formation and growth of cosmic objects is not an easy task. Even in the simple and yet most likely scenario, hereafter assumed, of structure formation via gravitational instability from a primordial Gaussian random field of density fluctuations with power spectrum leading to hierarchical clustering no exact model can be build. The reason for this is the lack of an exact solution for the growth of density fluctuations in the non-linear regime.

There are only two ways to circumvent this difficulty: the use of numerical simulations and the construction of (semi) analytical models relying on approximated collapse dynamics. The former is obviously more exact but it is not free of problems, either. Numerical simulations are very time-consuming, which translates into a limited dynamical range and a very reduced region of the parameter space covered. Moreover, numerical simulations give access to the yields of the complex processes taking place, but the full understanding of what is going on is not easy. In contrast, models are less accurate, sometimes possibly poorly justified, but are more practical and allow a deeper insight into the physics. In fact, both approaches are complementary: simulations ultimately justifies the

goodness of analytical models while the latter bring the possibility to comfortably explore a wide range of parameters and allow to better understand the results of the former. There are in the literature numerous reviews dealing with cosmological simulations. Here I will focus on the improvements achieved, for the last twenty years, in the construction of a detailed model for the hierarchical clustering of objects.

The different models developed so far are of two main kinds. On the one hand, there are models developed to derive the theoretical mass function of objects (or haloes). These are briefly reviewed in § 2, the most relevant ones being discussed in more detail in § 3. On the other hand, there are models which go further and provide us with typical times and rates of the clustering process. These latter models are addressed in § 4. For simplicity, I assume an Einstein-de Sitter ($\Omega = 1$, $\Lambda = 0$) universe and comoving units.

2. Theoretical Mass Functions

As mentioned, all clustering models are based on some approximation to the collapse dynamics of density fluctuations. Most of them, in particular the seminal model by Press & Schechter (1974; PS), rely on the spherical collapse model. This is a poor approximation, in general, to the real collapse. Yet, the PS mass function gives very good fits to N -body simulations (Nolthenius & White 1987; Efstathiou et al. 1988; Efstathiou & Rees 1988; Carlberg & Couchman 1989; White et al. 1993; Bahcall & Cen 1993; Lacey & Cole 1994). The reason is likely that massive objects, those intended to be described, arise from high amplitude peaks (density maxima) of the initial density field and the collapse of matter around such peaks is particularly well described by the spherical model (Bernardeau 1994). N -body simulations seem to show that there is no good correspondence between peaks and objects (van de Weygaert & Babul 1994; Katz, Quinn, & Gelb 1993). But this is likely due to a variety of effects, namely the nesting of peaks on different scales, the use of an unappropriated window, or the inclusion of density contrasts and masses which do not correspond to the collapse times and filtering scales analyzed (see below).

Actually, the PS mass function and new more or less sophisticated versions of it (Cole & Kaiser 1989; Bond et al. 1991, BCEK; Blanchard, Valls-Gabaud, & Mamon 1994; Jedamzik 1995; Yano, Nagashima, & Gouda 1995) do not explicitly deal with peaks as seeds of bound objects. But a parallel set of models has also been developed within the peak theory framework (Colafrancesco, Lucchin & Matarrese 1989; Bond 1989; Peacock & Heavens 1990; Apple & Jones 1990; Manrique & Salvador-Solé 1995) reaching similar results.

In the context of models relying on the spherical approximation, we must also mention the model constructed by Cavaliere & Menci (1994) using the theory of Cayley trees with disorder. This is a more general formalism which recovers, as two extreme limits, the diffusion equation describing, as shown by BCEK, the clustering of objects à la PS, and the Smoluchowski kinetic equation describing the aggregation of objects moving inside a relaxed system. Indeed, this formalism is intended to derive the mass function of objects accounting for the fact that they may survive and evolve inside larger scale objects (for example, galaxies in clusters). This mass function is different from that intended to be

derived in all previous models; these only consider relaxed haloes which are not embedded within any larger scale relaxed system. Here we will focus on the latter most usual viewpoint.

There are also a few models based on other dynamical approximations. Monaco (1995) has followed the PS approach but using the ellipsoidal collapse approximation. Bond & Myers (1993a, 1993b) have considered this latter approximation in the framework of the peak theory. Finally, Doroshkevich & Kotok (1990) and Vergassola et al. (1994) have used the adhesion model. In principle, these are better approximations to the true collapse than the simple spherical model. However, in the case of the adhesion approximation, the mathematical calculations are very complicated and one can only infer approximate analytical solutions for the cases of pure 1-D, 2-D, or 3-D collapsed structures (see Doroshkevich & Kotok 1990). For the real composite case, one can only obtain the asymptotical behavior (Vergassola et al. 1994). Concerning the mass function obtained by Monaco (1995), it is not clear why it does not recover the PS solution at the large mass end where the spherical approximation should be essentially correct. In fact, Bond & Myers (1993a, 1993b) find, in contrast, that the spherical collapse is a good approximation for very massive objects, indeed. The only drawback of the very accurate approach followed by these latter authors (the so-called “peak-patch” formalism) is that it involves complicated calculations including Monte-Carlo simulations which makes it less handy than usual (semi) analytical models.

3. Models based on the Spherical Collapse Approximation

3.1. The PS Mass Function

According to the spherical collapse model (Gunn & Gott 1972), the collapse time t for a shell of radius R around the center of a spherically symmetric, outwards decreasing (to avoid shell crossing until $\sim t$) linear density fluctuation partaking of the general Hubble expansion at t_i only depends on the mean density contrast δ (the density fluctuation normalized to the mean density of the universe) inside it through the relation $\delta(t) = \delta_{c0} a(t_i)/a(t)$, with $a(t)$ the cosmic expansion factor and δ_{c0} a constant equal to $3/20 (12\pi)^{2/3} \approx 1.69$. The collapse of that shell represents, of course, the appearance, at t , of a relaxed object of mass equal to (to 0th order in δ) $4\pi/3 \rho R^3$, with ρ the mean density of the universe.

Inspired of this simple model, PS assumed that any point in the initial (linear and Gaussian distributed) density field smoothed with a top-hat filter of scale R with density contrast *above* the overdensity δ_c collects matter so to reach, at t related to δ_c through the expression above, a mass *larger than* $M(R) = 4\pi/3 \rho R^3$. Consequently, by differentiating over M the volume fraction occupied by such points,

$$f(\geq \delta_c, R) = \frac{1}{2} \operatorname{erfc} \left[\frac{\delta_c}{\sqrt{2} \sigma_0(R)} \right], \quad (1)$$

with $\sigma_0(R)$ the rms density contrast on scale R , one should obtain the volume fraction contributing at t with objects of mass M to $M + dM$, and by dividing

it by M/ρ the number density of such objects

$$N(M, t) dM = 2 \frac{\rho}{M} \left| \frac{\partial f(\geq \delta_c, R)}{\partial R} \right| \frac{dR}{dM} dM. \quad (2)$$

It is worthwhile mentioning that, in the case of power-law power spectra, there should be no privileged time or scale (in an Einstein-de Sitter universe as assumed here). The PS mass function recovers this expected behavior. The number of objects in a volume M_*/ρ , with M_* corresponding to a scale defined through any arbitrary fixed value of $\sigma_0(R_*)$, with mass M/M_* in an infinitesimal range, as well as the volume (or mass) fraction subtended by objects of scaled mass M/M_* in an infinitesimal range are time invariant, indeed.

But the growth of density fluctuations can deviate from the spherical collapse in leaving the linear regime. Hence, one should check whether small changes in those aspects the most strongly connected with the spherical approximation are suitable. In particular, other filters than the top-hat one, and other values of constant δ_{c0} or of the proportionality factor q^3 between the mass and ρ times the natural volume of the filter should be investigated. (We must remark that there is degeneracy between the latter two constants, so there is just one degree of freedom for any given filter.) Yet, Lacey & Cole (1994) have recently shown that a very satisfactory fit to N -body data can be obtained for masses in the relevant range with a top-hat filter and δ_{c0} close to the standard value (for $q = 1$).

A more serious problem, apart from the unnatural seeds of bound objects assumed, concerns the unjustified factor two in the right-hand member of equation (2). This must be introduced for the final mass function to be correctly normalized, that is, for the integral of M times the mass function to be equal to the mean density of the universe. (Every particle in the universe is at any time t within some virialized object with appropriate mass.) On the other hand, the overcounting of objects actually swallowed by previously collapsed ones and the neglected contribution to the mass of objects of low density regions enclosed within high density ones (which might explain the fudge factor 2) are not accounted for. To analyze the effects of such cloud-in-cloud configurations Cole & Kaiser (1989) have devised a practical numerical method, called the “block model”. After decomposing (through a series of cuts in two pieces) a large cuboidal volume in very small cuboidal blocks with different overdensities (assigned at each level, through Monte Carlo, according to the Gaussian distribution corresponding at that scale) one can follow their detailed merger trees free of the cloud-in-cloud problem under the same clustering assumptions as in the PS approach (except for the rather unnatural geometry of the cuboidal filter).

3.2. The Excursion Set Formalism

A more satisfactory solution to these latter problems, in the sense that not attached to any particular realization (using a spherical filter) and leading to a fully analytical solution, was provided by BCEK by means of the powerful “excursion set” formalism. When the filter size is increased, the density contrast at a fixed point can diminish or increase depending on whether the original cloud is embedded in a higher density contrast one or not. So the random walk

followed by this point in the δ vs. R diagram will inform us on the nesting of clouds centered on that point. In particular, the mass of the only object which must be counted at t attached to a fixed point is given by the largest scale R for which the δ_c line is upcrossed.

The mathematical description of such random walks is hard to achieve in general. However, for the sharp k-space filter, the volumes subtended by different scales R are uncorrelated. Consequently, the random walk followed by $\delta(R)$ is then purely Brownian with variance $\sigma_0^2 = \langle R^2 \rangle \equiv S$ and the equation describing the number density $Q(S, \delta)$ of trajectories found at (S, δ) which start at (S_0, δ_0) is the simple diffusion equation

$$\frac{\partial Q}{\partial S} = \frac{1}{2} \frac{\partial^2 Q}{\partial \delta^2}. \quad (3)$$

Therefore, the volume fraction in objects with mass in the range M to $M + dM$, equal to the probability that a trajectory starting at $(S_0 = 0, \delta_0 = 0)$ (corresponding to the limit for $R = \infty$ of the smoothed density contrast attached to any fixed point) upcrosses for the first time δ_c in the corresponding range of S , is simply given by the reduction, in that range, in the number density of trajectories surviving below δ_c

$$f(\delta_c, S) dS = \left[-\frac{\partial}{\partial S} \int_{-\infty}^{\delta_c} Q(\delta, S) d\delta \right] dS, \quad (4)$$

with

$$Q(\delta, S) = \frac{1}{\sqrt{2\pi}} \left\{ \exp\left(-\frac{\delta_c^2}{2S}\right) - \exp\left[-\frac{(\delta - \delta_c)^2}{2S}\right] \right\} \quad (5)$$

the solution of the diffusion equation (2) with absorbing barrier at δ_c . Interestingly enough, the solution one gets (after changing to variable M) is just the PS mass function with the correct normalization factor 2. But, why the sharp k-space filter?

3.3. An Improved Correction for the Cloud-in-Cloud

Moreover, the previous formalism only corrects for nested configurations which are well centered on each fixed point; off-center nested configurations are not accounted for. To better correct for the cloud-in-cloud one must abandon the excursion set formalism. (This considers the evolution in the δ vs. R diagram of each fixed point separately, that is, it cannot see any correlation of the density field among different points.) Jedamzik (1995) proposed to directly apply the PS prescription, equation (2) to the volume fraction (1) uncorrected for any nesting, denoted here by subindex PS , minus the volume fraction in clouds nested within any larger scale cloud with δ_c

$$f(\geq \delta_c, R) = f_{PS}(\geq \delta_c, R) - \frac{1}{\rho} \int_R^\infty M(R') N(R', \delta_c) P(\geq \delta_c, R | \delta_c, R') dR. \quad (6)$$

In writing equation (6) we have taken into account the remarks by Yano, Nagashima, & Gouda (1995) on its correct expression. $P(\geq \delta_c, R | \delta_c, R')$ is the probability that a cloud of size R with $\delta \geq \delta_c$ is located on a background with

$\delta = \delta_c$ on scale R' , while $M(R') N(R', \delta_c) dR' / \rho$ approximately gives the probability that such a background is found inside a non-nested cloud with δ_c on scale in the range R' to $R' + dR'$. The probability P can be easily calculated in the case of sharp k-space filter, since the probability of finding two values of δ on different scales at a given point is then simply the product of finding each of them separately.

$N(R, \delta_c) dR$ is the unknown scale function, i.e., the mass function previous to the change to variable M , that we want to determine. Therefore, by applying the PS prescription to equation (e3) one is led to a Volterra type integral equation of the second kind for $N(M, t)$ which can be readily solved through the standard iterative method from the initial approximated solution given by the PS mass function. (This is equivalent to the practical algorithm proposed by Jedamzik to solve its equation.)

3.4. The PS Approach Extended to the Peak Model

But peaks are better motivated seeds of objects than the fuzzy regions considered in all previous models. Also inspired of the spherical collapse model, the “peak model” ansatz states that objects at a time t emerge from peaks with density contrast equal to a fixed linear overdensity δ_c in the smoothed, on any scale R , density field at the arbitrary initial time t_i . The critical overdensity is assumed to be a monotonous decreasing function of t , while the mass M of objects can also be assumed (the consistency of this guess is to be confirmed a posteriori) a monotonous increasing function of R .

The PS prescription (equation [1]) is therefore achieved, in this framework, by simply taking (Colafrancesco, Lucchin, & Matarrese 1989; Peacock & Heavens 1990)

$$f(\geq \delta_c, R) = n_{pk}(\delta_c, R) \frac{M_{pk}(\delta_c, R)}{\rho}, \quad (7)$$

where $n_{pk}(\delta_c, R)$ is the number density of peaks with $\delta \geq \delta_c$ in the density field smoothed on scale R , calculated by Bardeen et al. (1986; BBKS), and $M_{pk}(\delta_c, R)$ is the average mass of their respective collapsing clouds, i.e., of the objects giving them rise. Note that since peaks in $n_{pk}(\delta_c, R)$ do not have, in general, $\delta = \delta_c$, the average mass of their collapsing clouds, $M_{pk}(\delta_c, R)$, will differ from $M(R)$. The above mentioned problems with the normalization of the PS mass function and the cloud-in-cloud are reflected in the different expressions for $M_{pk}(\delta_c, R)$ found in the literature.

But there is a more serious problem. In applying equation (2) to the volume fraction (7) it has been implicitly assumed that: 1) the total mass in collapsing clouds associated with peaks (with $\delta > 0$) is conserved with varying scale, and 2) the density contrast of peaks is a decreasing function of scale. This guarantees, indeed, that the variation along dR of the mass associated with peaks above δ_c is just that associated with peaks crossing δ_c in that infinitesimal range of scales. Both points seem to follow from the peak model ansatz, but they actually do not. As shown below, point 2 crucially depends on the shape of the filter used, while mergers invalidate point 1 in any event.

3.5. An Extension to Peaks Inspired of the Excursion Set Formalism

As pointed out by Bond (1988), the only reliable strategy to derive the mass function in the peak model framework is therefore to directly count the density of peaks with density contrast upcrossing δ_c in an infinitesimal range of scale, $N_{pk}(R, \delta_c) dR$, then correct for the cloud-in-cloud, and finally transform to the mass function of objects at t , $N(M, t) dM$, through the appropriate $M(R)$ and $\delta_c(t)$ relations.

Taking into account that for a Gaussian filter we have

$$\frac{d\delta}{dR} = R \nabla^2 \delta, \quad (8)$$

Bond (1989) and Appel & Jones (1990) derived the wanted scale function of peaks by computing, à la BBKS, the density of peaks at R with the extra constraint that they cross δ_c between R and $R + dR$,

$$\delta_c < \delta \leq \delta_c - \nabla^2 \delta R dR. \quad (9)$$

This leads to

$$N_{pk}(R, \delta_c) dR = \left. \frac{dn_{pk}(\nu, R)}{d\nu} \right|_{\nu=\delta_c/\sigma_0} \frac{\sigma_2(R)}{\sigma_0(R)} <x> R dR, \quad (10)$$

where $n_{pk}(\nu, R)$ is the density appearing in equation (7), with $\nu \equiv \delta/\sigma_0(R)$, and $<x>$ is an average of $-\nabla\delta/\sigma_2(R)$, with $\sigma_2(R)$ the second order spectral moment, given in Manrique & Salvador-Solé (1995a; MSa).

To correct this scale function for the cloud-in-cloud Bond (1989) used the approximate exclusion factor

$$F(R, \delta_c) = \exp \left[- \int_R^\infty dR' \frac{M(R')}{\rho} N_{pk}(R', \delta_c) \right] \quad (11)$$

obtained from the excursion set formalism. Note that this coincides with the Poisson probability that, in a volume typically harboring one peak on scale R , there is no such peak located in the volume fraction independently subtended by collapsing clouds associated with larger scale peaks.

But what $M(R)$ and $\delta_c(t)$ relations must we take to transform this corrected scale function to the wanted mass function, and why should we use the Gaussian filter? And what is worse, the previous derivation implicitly assumes that the spatial location of peaks does not change in varying the filtering scale which is obviously not true in general.

3.6. The Confluent System Formalism

To account for this variation MSa have developed a new formalism, the “confluent system of peak trajectories”, able to follow the filtering evolution of peaks despite their spatial shift.

To guarantee that one peak on scale $R + \Delta R$, with ΔR arbitrarily small, traces the same accreting object as another peak on scale R at the times corresponding to their respective density contrasts, the separation between both

points must be, at most, of the order of ΔR . In this manner, the collapsing cloud associated with the peak on scale $R + \Delta R$ will essentially include the whole cloud associated with the peak on scale R . Furthermore, this proximity condition is not only necessary, but also sufficient: as readily seen from the Taylor series expansion of the density gradient around a density maximum, there cannot be more than one peak on scale $R + \Delta R$ in the close neighborhood of any peak on scale R . This identification allows one to draw a δ vs. R diagram similar to the excursion set one but for the fact that each trajectory $\delta(R)$ is now attached to one individual accreting object, i.e., to *the changing peaks tracing it* in the filtering process, instead of to one fixed point. It is shown that the total derivative $d\delta/dR$ of a peak trajectory in this diagram coincides with the partial derivative $\partial_R \delta$ of the current peak. Moreover, for the mass of accreting objects to increase with time, δ must decrease with increasing R along any peak trajectory, which is only satisfied *for a Gaussian filter*.

The density of peak trajectories upcrossing the δ_c line in an infinitesimal range of scales is equal to the density of peaks on scale R with $\delta \geq \delta_c$ *evolving* into peaks with $\delta \leq \delta_c$ on scale $R + \Delta R$. Given the mandatory Gaussian filter and the form of the total derivative of δ over R along a peak trajectory one is just led to equations (e8)–(e10). The important point is that this derivation is now fully justified.

Moreover, to correct for the cloud-in-cloud MSa followed the more accurate approach pointed out by Jedamzik (1994). The result is the Volterra integral equation of the second kind

$$N(R, \delta_c) = N_{pk}(R, \delta_c) - \frac{1}{\rho} \int_R^\infty dR' M(R') N(R', \delta_c) N_{pk}(R, \delta_c | R', \delta_c). \quad (12)$$

In equation (12) $N_{pk}(R, \delta_c | R', \delta_c) dR$ is the conditional density of peaks with δ_c on scales R to $R + dR$ given that they have density δ_c on scale R' , which can be written in terms of the analog density per infinitesimal density contrast calculated by BBKS in a similar way as equation (10), and $\rho^{-1} M(R') N(R', \delta_c) dR'$ gives the approximate probability to find such a point inside the collapsing cloud associated with a non-nested peak with δ_c on some scale in the range R' to $R' + dR'$.

Now, if the density field is endowed with a power-law power spectrum the scale function must be self-similar. Likewise, the mass fraction in objects with scaled mass M/M_* in an infinitesimal range, as well as the number of peaks inside the volume M_*/ρ on scales R/R_* in an infinitesimal range must be invariant. But this is only satisfied provided $M(R) = \rho (2\pi)^{3/2} [q R]^3$, with $(2\pi)^{3/2} R^3$ the natural volume associated with the Gaussian window and q an arbitrary constant. On the other hand, the mass function at t is independent of the arbitrary initial time t_i provided only $\delta_c(t) = \delta_{c0} a(t_i)/a(t)$ with δ_{c0} an arbitrary constant. (In contrast with the PS case, there is no degeneracy now between constants q and δ_{c0} .) With these relations the scale function (12) leads to the wanted mass function which turns out to be correctly normalized for whatever values of q and δ_{c0} governing the exact collapse dynamics. A good fit can be obtained to the PS mass function at any time t for appropriate values of these parameters. For non-power-law spectra, the previous $M(R)$ and $\delta_c(t)$ relations are shown to also approximately hold. In this case, however, there is

one unique value of q yielding the correct normalization for whatever value of δ_{c0} . Nonetheless, a good fit can also be obtained to the corresponding PS mass function at any time for an appropriate value this parameter.

4. Growth Rates and Times

Richstone, Loeb, & Turner (1992) proposed the time derivative of the PS mass function as an estimate of the formation rate of objects of mass M at a given epoch. However, this is a very crude estimate since that quantity is actually equal to the rate at which objects reach mass M *minus the rate at which they leave this state*, both terms having comparable values.

4.1. The Excursion Set Formalism

Following the PS original approach Bower (1991) derived the conditional mass function of objects of mass at some epoch subject to being part of another object with a given larger mass at a later time. This was subsequently achieved by BCEK from the excursion set formalism. To do it one must simply compute the volume fraction in objects with S in an infinitesimal range (4) given by the solution of the diffusion equation (3) with barrier δ_c now with initial condition ($S_0 = S', \delta_0 = \delta'_c$) corresponding to the more massive object at the later epoch, instead of (0,0),

$$f(S, \delta_c | S', \delta'_c) dS = \left\{ \frac{\delta_c - \delta'_c}{\sqrt{2\pi} (S - S')^{3/2}} \exp \left[\frac{(\delta_c - \delta'_c)^2}{2(S - S')} \right] \right\} dS, \quad (13)$$

and proceed in the usual manner.

The resulting conditional mass function $N(M, t | M', t') dM$ was used by Lacey & Cole (1993; LC; see also Kauffmann & White 1993) to infer the instantaneous merger rate of objects of mass M at t into objects of mass M' to $M' + dM'$

$$r^m(M \rightarrow M', t) dM' = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \frac{N(M, t | M', t + \Delta t) N(M', t + \Delta t) dM'}{N(M, t)}. \quad (14)$$

This clustering model has been shown by Lacey & Cole (1994) to be in very good agreement with N -body simulations. However, the PS approach is not fully satisfactory (see § 3). On the other hand, accretion does not play any role in this model; one can only follow the instantaneous mass increase of objects, event which is generically called merger. As a consequence, there is no specific event marking the beginning or the end of any entity, hence, properly justifying the words formation or destruction of objects. This is the reason why the age and survival time of any object must be defined in terms of the relative variation (say, by a factor 2) in mass along the series of objects with embedded mass connecting with it.

4.2. The Confluent System Formalism

The model based on the confluent system formalism is better justified (peaks are the seeds of objects and simple consistency arguments unambiguously fix

the filter and the $M(R)$ and $\delta_c(t)$ relations to be used) and makes the effective distinction between merger and accretion.

When an object evolves by accretion (tracing a continuous curve $\delta(R)$ in the δ vs. R diagram) the volume M/ρ of the collapsing cloud associated with the corresponding evolving peak increases. This makes smaller scale peaks to become nested within it. Their $\delta(R)$ curves experience then a discontinuity in R at a fixed δ which can be naturally interpreted as a merger. The net density of peaks with δ on scales R to $R + dR$ becoming nested in non-nested peaks with $\delta - d\delta$ on scales R' to $R' + dR'$, $\mathbf{N}^d(R \rightarrow R', \delta) dR dR' d\delta$, can then be accurately calculated (Manrique & Salvador-Solé 1995b; MSb). The instantaneous (true) merger or destruction rate at t for objects of mass M per specific infinitesimal range of mass M' ($M < M'$) of the resulting objects is, therefore,

$$r^d(M \rightarrow M', t) = \frac{\mathbf{N}^d(R \rightarrow R', \delta_c)}{N(R, \delta_c)} \frac{dR'}{dM'} \left| \frac{d\delta_c}{dt} \right|. \quad (15)$$

Note that this merger rate is different from that obtained by LC because, in the latter, captures of infinitesimal objects are included while, in the former, they are not.

In addition, objects forming in the interval of time dt from the merger of similarly massive objects are traced by peaks appearing (there is no previous peak to be identified with) in the corresponding range of density contrasts $-d\delta$ without being nested. The net density of non-nested peaks appearing between δ and $\delta - d\delta$, $\mathbf{N}^f(R, \delta) dR d\delta$, can also be calculated (MSb). This leads to the instantaneous formation rate at t of objects of mass M

$$r^f(M, t) = \frac{\mathbf{N}^f(R, \delta_c)}{N(R, \delta_c)} \left| \frac{d\delta_c}{dt} \right|. \quad (16)$$

Finally, the instantaneous mass accretion rate of objects of mass M follows from the instantaneous scale increase rate of the corresponding peaks as they evolve along continuous trajectories in the δ vs. R diagram. From equation (8) we have $dR/d\delta = [-x R \sigma_2(R)]^{-1}$. Averaging over the scaled Laplacian of each peak x leads to (MSb)

$$r_{mass}^a(M, t) = \frac{1}{\langle x \rangle \sigma_2 R} \frac{dM}{dR} \left| \frac{d\delta_c}{dt} \right|. \quad (17)$$

On the other hand, the density $N_{sur}(t) dM$ of objects surviving (i.e., having not merged, just accreted) until the time t from a typical population of objects with masses in the range M_0 to $M_0 + dM$ at $t_0 < t$ is given by the solution, for the initial condition $N_{sur}(t_0) = N(M_0, t_0)$, of the differential equation

$$\frac{dN_{sur}}{dt} = -r^d[M(t), t] N_{sur}(t) \quad (18)$$

with $r^d[M(t), t]$ the integral over M' of the specific merger rate (15). Hereafter, $M(t)$ is the typical mass at t of such accreting objects, approximately given by the solution of

$$\frac{dM}{dt} = r_{mass}^a[M(t), t], \quad (19)$$

with $M(t_0) = M_0$. Hence, by defining the typical survival time, $t_{sur}(M_0, t_0)$, of objects with masses M_0 to $M_0 + dM$ at t_0 as the interval of time since t_0 after which their density is reduced (owing to mergers) by a factor e , we are led to the equality $t_{sur} = t_d - t_0$, where the destruction time $t_d(M_0, t_0)$ is given by the implicit equation $N_{sur}(t_d) = e^{-1} N(M_0, t_0)$.

Likewise, the density $N_{pre}(t) dM$ of objects at t_0 that already existed (i.e., they have just accreted matter since) at a time $t < t_0$ is given by the solution of

$$\frac{dN_{pre}}{dt} = r^f[M(t), t] N[M(t), t] - r^d[M(t), t] N_{pre}(t), \quad (20)$$

with $N_{pre}(t_0) = N(M_0, t_0)$. Thus, by defining the typical age $t_{age}(M_0, t_0)$ of objects with masses between M_0 and $M_0 + dM$ at t_0 as the interval of time until t_0 before which their density (owing to their progressive formation and possible disappearance) was a factor e smaller, we are led to the equality $t_{age} = t_0 - t_f$, where the formation time $t_f(M_0, t_0)$ is given by the solution of the implicit equation $N_{pre}(t_f) = e^{-1} N(M_0, t_0)$.

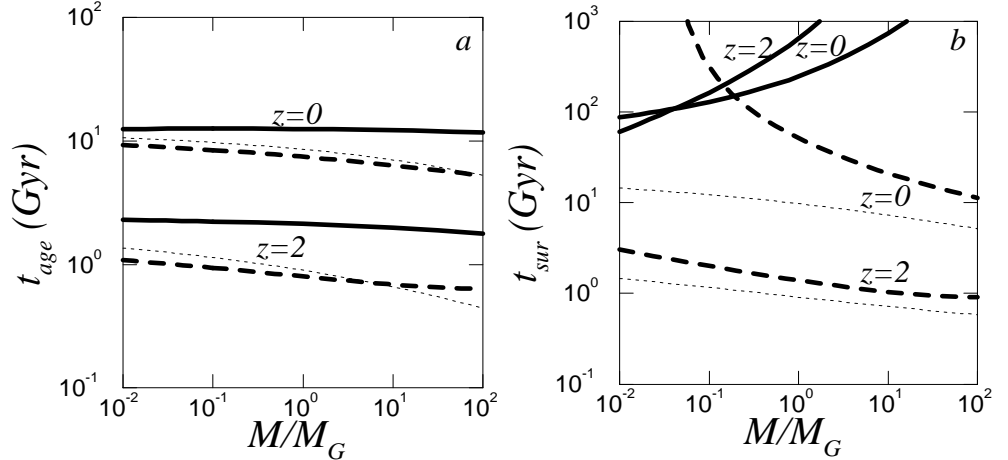


Figure 1. Age (a) and survival time (b) for objects in the CDM cosmology with $\sigma_8 = 1.5$ ($M_G = 10^{12} M_\odot$). Results obtained by MSb with the half an double-mass-accretion times in dashed lines (thick) and LC (thin).

One can also define similar times as those adopted by LC as an estimate of the age and survival time of objects. These are called half-mass-accretion time and double-mass-accretion and are defined as the interval of time spent since the mass of an object was typically half its current value and the interval of time required by an object to typically double its mass, respectively. (The only difference from the analog times defined by LC is that, since the new times refer to the typical mass evolution of *given objects*, they only involve accretion.) These time estimates can be readily obtained from equation (19). In Figure 1 we plot for comparison the three sets of characteristic times for objects of different masses at two different epochs.

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